SEM-2(MATHEMATICS GENERAL)

ANUP RAY

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Date

Ex: Solve $x^2p^2 + xyp - 6y^2 = 0$

Ans: We have $x^2p^2 + xyp - 6y^2 = 0$

or $x^2p^2 + 3xyp - 2xyp - 6y^2 = 0$

or $px(px + 3y) - 2y(px + 3y) = 0$

or $(px + 3y)(px - 2y) = 0$

Either $px + 3y = 0 \Rightarrow x\frac{dy}{dx} + 3y = 0$

or $\frac{dy}{y} + 3\frac{dx}{x} = 0$

Integrating we get, $\log y + 3\log x = \log c$, where c being arbitrary constant.

or $yx^3 - c = 0$

otherwise $px - 2y = 0$ or $x\frac{dy}{dx} - 2y = 0$

or $\frac{dy}{y} - 2\frac{dx}{x} = 0$

Integrating we get, $\log y - 2\log x = \log c$

or $y - cx^2 = 0$

∴ The general solution is

$(yx^3 - c)(y - cx^2) = 0$

Ex: Solve $p^2 + \left(x + y - \frac{2y}{x}\right)p + xy + \frac{y^2}{x^2} - y - \frac{y^2}{x} = 0$

Ans: The given equation can be written as

$p^2 + \left[\left(x - \frac{y}{x}\right) + \left(y - \frac{y}{x}\right)\right]p + \left(x - \frac{y}{x}\right)\left(y - \frac{y}{x}\right) = 0$

or $p\left(p - \frac{y}{x} + x\right) + \left(y - \frac{y}{x}\right)\left(p - \frac{y}{x} + x\right) = 0$

or $\left(p + x - \frac{y}{x}\right)\left(p + y - \frac{y}{x}\right) = 0$

Either $p + x - \frac{y}{x} = 0$ or $\frac{dy}{dx} - \frac{1}{x} \cdot y = -x$ $\rightarrow (1)$

which is a linear equation.

Its I.F. $= e^{\int -\frac{1}{x}\,dx} = e^{-\log x} = \frac{1}{x}$

Multiplying both sides of (1) by I.F. we get

$$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = -1$$

or $\frac{d}{dx}\left(\frac{y}{x}\right) = -1$

Integrating we get, $\frac{y}{x} = -x + c$

otherwise $p + y - \frac{y}{x} = 0$ or $\frac{dy}{dx} + y\left(1 - \frac{1}{x}\right)$

$\Rightarrow \frac{dy}{y} + \left(1 - \frac{1}{x}\right)dx = 0$

Integrating $\log y + x - \log x = \log c$

or $y = cx e^{-x}$

Hence the general solution is

$\left(\frac{y}{x} + x - c\right)\left(y - cx e^{-x}\right) = 0$

Ex: Solve the following equations: (Home work)

a) $p^2 + 2px + py + 2xy = 0$ b) $p^2 - 7p + 12 =$

c) $p^2 + px = xy + y^2$ d) $p^2 + 2px - 3x^2 = 0$

e) $p^2 - p(e^x + e^{-x}) + 1 = 0$ f) $p^2 y - p(xy + 1) + x =$